FM2 - I CC1

MATRIZEAK - MATRIZE BEREIZIBATUAK

- Ekuazioketako eragiketak $A \rightsquigarrow B$ [Eragiketen ostean ekuazioak dira]
- Matrize antzekoak

matrizearen ordena: $A \in M_{m \times n}(\mathbb{R})$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

ekuazioak → E_i (lerroak)
m → ekuazioak
n → zutabeak

DETERMINANTE - KALKULUAK - TRANSFORMAZIOAK

- Metodoak $\begin{cases} n=3 \to \text{Sarrus} \\ n>3 \end{cases}$ $\underbrace{\sum_{j=1}^{n} a_{ij}A_{ij}^{*} \ , \ \sum_{i=1}^{n} a_{ij}A_{ij}^{*}}$
- Matrizeak kanonikoak bihurtuz ditezen
- Elementu bat izan ezik, ekuazio edo zutabe batean, gainerako elementuak zero direla
- Adjuntua → $A_{ij} = (-1)^{i+j} M_{ij} = \pm M_{ij}$

Ordenagailuak erabiliz ez dira...

*₁ Determinantea lortzeko lerro edo zutabe bateko elementuak zein beraien adjuntuekin biderkatu eta batuz, emaitza lortzen dugu.

|Konbinazio linealak|

- $a, b, c \dots \in \mathbb{R}$
  $aE_1 + bE_2 + \dots$
- $E_i \pm E_j$
  $2E_i + E_j$

! Bi ekuazio berdinak badaude, determinantearen balioa zero da.

|Eragiketak|

1) $E_i \leftrightarrow E_j$ ekuazioen artean trukaketa → $\det A \Rightarrow -\det A$
2) $\forall \lambda \in \mathbb{R}$; $|\lambda E_i|$ ekuazioaren bat zenbakiarekin biderkatu → $\det A \Rightarrow \lambda \cdot \det A$] determinantea aldatu egiten da
3) $E_i \to E_i \pm [\lambda_i E_j + \lambda_n E_k + \dots]$ beste ekuazio bat gehitzea/kentzea (konbinazio lineala) → $\det A = \det A$ determinantea ez da aldatzen

|Rouche - Frobenius|

Heineren araberako sailkapena

- $h(A)$ → Matrize itxia
  $h(A_r)$ → Matrize zabaldua

$h(A) = h(A_r)$ $\begin{cases} = n \text{ Bateragarri determinatua} \\ < n \text{ Bateragarri indeterminatua} \end{cases}$

$h(A) \neq h(A_r)$ Bateraezina

|Matrize alderantzigarriak|

- $A \in M_{n \times n}(\mathbb{R}) \,/\, \exists B \in M_{n \times n}(\mathbb{R})$
  $A \cdot B = B \cdot A = I_n$ $B = A^{-1}$

matrize singularra → $\det = 0$
! Ez da alderantzigarria

- A → matrize erregularra ($\det \neq 0$)

- Kalkuluak
  1) $A^{-1} = \frac{1}{\det(A)} \tilde{A}^{t}$ $\quad \tilde{A} = (A_{ij})^{*}$ Matrize adjuntua
  2) Ekuazioetako eragiketak → $A | I_n \to I_n | A^{-1}$
  3) EIS bateratzea
  4) $P(A) = 0$

$$A = \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{c} \frac{1}{2}C_1 \\ \frac{1}{3}C_2 \\ \frac{1}{4}C_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/4 \end{array}\right)$$

$$\downarrow$$

$$A^{-1}$$

$\boxed{\text{2) procedura}}$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}^T = \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}}_{\tilde{A}} \frac{1}{2} = \underbrace{\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix}}_{A^{-1}}$$

$\boxed{\text{1) procedura}}$

$$A^3 - 3A^2 + 4A - 2I_3 = [0]_{3\times 3}$$

- I_3 [illegible]
- A [illegible]
- $A[\quad] = I_3$
- A^{-1} [illegible]

$$\frac{A^3 - 3A^2 + 4A}{2} = I_3 \qquad A\left[\frac{1}{2}(A^2 - 3A + 4I)\right] = I_3$$

$$2A^{-1} = A^2 - 3A + 4I \qquad A^{-1} = \frac{1}{2}[A^2 - 3A + 4I]$$

$\boxed{\text{4) procedura}}$

[illegible]

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad A^2 = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 & -3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix} = A^{-1}$$

$\boxed{\text{[illegible]}}$

$$A \cdot B = C \Rightarrow A \cdot B \cdot B^{-1} = C \cdot B^{-1} \Rightarrow A = C \cdot B^{-1}$$

$$\Rightarrow A^{-1} A \cdot B = A^{-1} C \Rightarrow B = A^{-1} C$$

$$A \cdot B \cdot C = D \Rightarrow A(BC) = D \Rightarrow A(BC)(BC)^{-1} = D(BC)^{-1} \Rightarrow A = D(BC)^{-1}$$

$$\Rightarrow A^{-1} A\, B\, C\, C^{-1} = A^{-1} D\, C^{-1} \Rightarrow B = A^{-1} D C^{-1}$$

$$\Rightarrow (AB)C = D \Rightarrow (AB)(AB)^{-1} C = (AB)^{-1} D \Rightarrow C = (AB)^{-1} D$$

$$(A \cdot B)^T = B^T A^T \qquad (A^T)^T = A$$

$$(A \cdot B)^{-1} = B^{-1} A^{-1} \qquad (A^{-1})^{-1} = A$$

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$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$A \cdot x = y$

$x = A^{-1} y$

$\boxed{3)\ \text{proxed a}}$

$$\begin{cases} x_1 - x_3 = y_1 \\ x_2 = y_2 \\ x_1 + x_3 = y_3 \end{cases}$$

$$A/At = \left(\begin{array}{ccc|c} 1 & 0 & -1 & y_1 \\ 0 & 1 & 0 & y_2 \\ 1 & 0 & 1 & y_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{y_1+y_3}{2} \\ 0 & 1 & 0 & y_2 \\ 0 & 0 & 1 & y_3 - \frac{y_1+y_3}{2} \end{array}\right)$$

$x_1 = \frac{y_1+y_3}{2}$

$x_2 = y_2$

$x_3 = y_3 - \left(\frac{y_1+y_3}{2}\right) = y_3 - x_1 = \frac{1}{2}y_3 - \frac{1}{2}y_1$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix}}_{A^{-1}} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Ortogonala $\Rightarrow A^T = A^{-1}$

$\det(A) = \det(A^T)$

$\det(A^T)\det(A) = 1$

$\det(A)^2 = 1$

$\det A = \pm 1$

$\det(A) = \det(A^T)$

$\det(AB) = \det(A)\det(B)$

$\det(A) = \kappa \quad \det(A^{-1}) = \frac{1}{\kappa}$

Idempotentea $\rightarrow A^2 = A$

Nilpotentea $\rightarrow \det = 0$

$(A+B)^3(A-B) = A-B \Rightarrow (A+B)^2(A+B)(A-B) = A-B \Rightarrow (A+B)^2(A^2-B^2) = A-B \Rightarrow (A+B)^2(A-B) = A-B$

$(A^3 + A^2B + AB^2 + B^3)(A-B) = A-B$

$(A + AB + B)(A-B) = A-B$

$A + AB + 2AB - 2AB + BA - B = A-B$

$\boxed{A-B = A-B}$

$(A^2 + B^2 + 2AB)(A^2 - B^2)$

$A - AB + AB - B + 2AB - 2AB = A-B$

$A - B = A - B$

$(A+B)(A+B)(A-B) = A-B$

$(A+B)(A^2-B^2) = A-B$

$(A+B)(A-B) = A+B$

$(A^2-B^2) = A-B$

$A-B = A-B$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{11}A_{11} + A_{12}A_{12} & A_{11}A_{21} + A_{12}A_{22} \\ A_{21}A_{11} + A_{22}A_{12} & A_{21}A_{21} + A_{22}A_{22} \end{pmatrix}$$

$(A A^T)$ simetrica $\quad (A A^T)^T = (A^T)^T A^T = A A^T$

A ele B simetrica $\Rightarrow (A+B)^T \quad A = A^T \quad B = B^T$

$(AB)^T \cdot (AB)^T$

FM2-II004

$A^p = [0]_{n\times n}$ A singularra

$[\text{Det} A]^p = 0$ $|A| = 0$

$\Downarrow$

$\text{Det} A = 0$

A ortogonala

$A^T = A^{-1} \Rightarrow A \cdot A^T = A^T \cdot A = I$

$A\, A^{-1} = A^{-1} A = I$

$A \cdot A^T = A^T \cdot A$

$P = A^{-1} A^T$ ortogonala dela frogatu

$P\, P^T = I$, $A^{-1} A^T P^T = \boxed{\cancel{A^{-1}A^T \cdot (A^{-1}A^T)^T}} = A^{-1} A^T (A^T)^T \cdot (A^{-1})^T = A^{-1} A^T A \cdot (A^T)^{-1} = A^{-1} A \cdot A^T (A^T)^{-1}$

$(A^{-1} A^T)^T$ $(A \cdot B)^T = B^T \cdot A^T$

$I \cdot I = I$

ERAGIKETAK ALDERANTZIZKOEKIN

$A \cdot B = C \rightarrow A B B^{-1} = C \cdot B^{-1} \rightarrow A = C B^{-1}$

$\rightarrow A^{-1} A B = A^{-1} C \rightarrow B = A^{-1} C$

$A B C = D \rightarrow A (BC) = D \rightarrow A (BC)(BC)^{-1} = D (BC)^{-1} \rightarrow A = D (BC)^{-1}$

$\rightarrow A^{-1} A B C C^{-1} = A^{-1} D C^{-1} \rightarrow B = A^{-1} D C^{-1}$

$\rightarrow (AB) C = D \rightarrow (AB)(AB)^{-1} C = (AB)^{-1} D \rightarrow C = (AB)^{-1} D$

$(AB)^T = B^T A^T$ $(A^T)^T = A$

$(AB)^{-1} = B^{-1} A^{-1}$ $(A^{-1})^{-1} = A$

$\det (AB) = \det A \cdot \det B$ $A A^{-1} = A^{-1} A = I$

$\det A = 3$

$\det A^{-1} = ?$

$(\det A^{-1})(\det A) = \det I$

$\det A^{-1} \cdot 3 = 1$ $\det A^{-1} = 1/3$

$A^T = A^{-1} \rightarrow$ Matrize ortogonala

$\det A$?

$A\, A^T = I$ $\det A^T \det A = 1$

$\det A = \det A^T$

$(\det A)^2 = 1$ $\det A = \pm 1$

FM2 . 5 - 005

1. ketek

$A, B \in M_{n\times n}(\mathbb{R})$ / $A\cdot B = B\cdot A$

Idenpotenteak $\begin{cases} A^2 = A \\ B^2 = B \end{cases}$

Frogatu: $(A+B)^3 (A-B) = A-B$

$$(A+B)^2 (A+B)(A-B) = A-B$$

$$(A^2+2AB+B^2)(A^2-B^2) = A\cdot B$$

$$(A+2AB+B)(A-B) = A-B$$

$$\cancel{A^2} - \cancel{AB} + \cancel{2A^2B} - \cancel{2AB^2} + \cancel{AB} - B^2 = A-B$$

$$\boxed{A-B = A-B}$$

$A, B \in M_{n\times n}(\mathbb{R})$ Frogatu:

- $A\,A^T$ simetrikoa dela

$$A\,A^T = (A\,A^T)^T = A^T \cdot (A^T)^T = A^T \cdot A$$

- A eta B simetrikoak $\Rightarrow (A+B)^n$ simetrikoa $\forall n \in \mathbb{N}$

$A = B^T \quad B = A^T$

$$(A+B)^n = (B^T+A^T)^n = [(B+A)^T]^n = [(A+B)^n]^T$$

- A eta B antisimetrikoak $\rightarrow \forall n$ bikoiti $\in \mathbb{N} \rightarrow (A+B)^n$ simetrikoa
$\forall n$ bakoiti $\in \mathbb{N} \rightarrow (A+B)^n$ antisimetrikoa

$\cancel{A^T} = B \quad B^T = A$

$A = -A^T \rightarrow -A = A^T$

$B = -B^T \rightarrow -B = B^T$

$$[(A+B)^n]^T = \{[-1(A^T+B^T)]^n\} = (-1)^n [(A+B)^{T\cdot n}]^T$$
$$= (-1)^n (A+B)^n$$

$$[(A+B)^n]^T = \begin{cases} n = 2k \rightarrow (A+B)^n \\ n = 2k-1 \rightarrow -(A+B)^n \end{cases}$$

- A eta B antisim.: $A\cdot B$ sim $\Leftrightarrow A\cdot B = B\cdot A$

$A^T = -B \quad B^T = -A \quad (A\,B)^T = [(-B^T)\cdot(-A^T)]^T = (B^T A^T)^T = [(B\,A)^T]^T = B\cdot A$

$A\cdot B$ sim $\Rightarrow (A\,B)^T = A\,B$

$\Big\}\; A\cdot B = B\,A$

FM2-J-006

$A \in M_{n\times n}(\mathbb{R})$ ezarrein matrize nilpotente singularra dela frogatu.

$A^p = [0]_{n\times n}$ $\quad \det(A^p) = \det[0]_{n\times n} = 0$

$(\det A)^p = \det[0]_{n\times n}$ $\quad (\det A)^p = 0 \to \det A = 0$

V + . . . → Kanpo konposaketa legea

Barneko konposaketa legea

$\forall \bar u, \bar v \in V$

$\bar u + \bar v \in V$

$\forall \lambda \in \mathbb{R} \longrightarrow \lambda \bar u \in V$

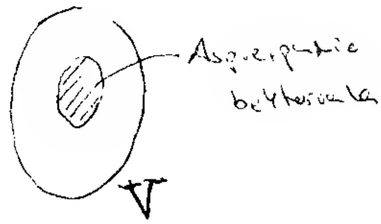
$\forall \lambda \notin \mathbb{K}$

$\lambda \cdot \bar u + \mu \cdot \bar v \in V$

Konbinazio lineala

V:

- $\mathbb{R}^n$, $n \in \mathbb{N}$: $\mathbb{R}^2 \Rightarrow (a,b)$ $a,b \in \mathbb{R}$; $\mathbb{R}^3 \Rightarrow (a,b,c)$ $a,b,c \in \mathbb{R}$; $\mathbb{R}^4 \Rightarrow (a,b,c,d)$ $a,b,c,d \in \mathbb{R}$
- P_n; $\forall \bar u \in P_n / \bar u = P_n(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$
- $M_{n\times n}(\mathbb{R})$
- C, $[a,b]$ jarraitua



F: Bektore-sistema → Sistema lotua / Sistema askea

Bektore-sistema baten heina: $h(F)$

Adib.

$F = \{(1,2)\ (2,3)\} \subset \mathbb{R}^2$ → bektore espazioa

$h(F) = \begin{pmatrix}1&2\\2&3\end{pmatrix} = 2$ → Zenbat bektore linealki independente dituzu $\to h(F) = 2 = m_K \to$ sistema askea

$G = \{(1,2),(2,4)\} \subset \mathbb{R}^2$

$h(G) = \begin{pmatrix}1&2\\2&4\end{pmatrix} = 1 \Rightarrow h(G) = 1 < m_K \to$ sistema lotua

m_K: bektore kopurua

$M = \{x^2, 3-2x, x^2+2x-3\} \subset P_2 \Rightarrow M = \{(0,0,1),(3,-2,0),(-3,2,1)\}$

$h(M) = \begin{vmatrix}0&0&1\\3&-2&0\\-3&2&1\end{vmatrix} = 2 \to h(M) = 2 < m_K \to$ sistema lotua

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$$H=\left\{\begin{pmatrix}2&3\\0&0\end{pmatrix},\begin{pmatrix}-3&0\\1&0\end{pmatrix},\begin{pmatrix}7&6\\1&0\end{pmatrix}\right\}$$

$$H'=\{(2,3,0,0),(-3,0,1,0),(7,6,-1,0)\}$$

$$h(H)=\begin{pmatrix}2&3&0&0\\-3&0&1&0\\7&6&1&0\end{pmatrix}=2<m_{ex} \quad \text{sistema lotua}$$

12 Ariketa

a) Azpiespazio bektorialak dira?

$S=\{(0,a,b)\ /\ a,b\in\mathbb{R}\}\subset\mathbb{R}^3$

$(0,a,b)=(0,a,0)+(0,0,b)=a(0,1,0)+b(0,0,1)$

$\vec{v}\in S.\quad \vec{u}=(0,a_1,b_1)$
$\quad\quad\quad \vec{v}=(0,a_2,b_2)$

$\vec{u}+\vec{v}=(0,a_1+a_2,b_1+b_2)\subset S?$ Bai, beraz azpiespazio bektoriala.

$T=\{(0,a,1)\ /\ a\in\mathbb{R}\}$

$\vec{u}=(0,0,1)\in T$
$\vec{v}=(0,1,2)\notin T$
$\vec{w}=(0,2,1)\in T$

$\vec{v}+\vec{w}=(0,2,2)\notin T$

$\vec{u}=(0,a_1,1)\ ;\ \lambda\vec{u}=\lambda(0,a,1)$
$\vec{v}=(0,a_2,1)$

$\vec{u}+\vec{v}=(0,a_1+a_2,2)\notin T$
Ez da azpiespazio bektoriala

$Z=\{(x,y,z)\ /\ x+y=0\ \wedge\ x-y=z\}$

$\vec{u}=(x_1,y_1,z_1)$
$\vec{v}=(x_2,y_2,z_2)$

$\vec{u}+\vec{v}=(\underbrace{x_1+x_2}_{x_b},\underbrace{y_1+y_2}_{y_b},\underbrace{z_1+z_2}_{z_b})\in Z\begin{cases}x_b+y_b=0\\x_b-y_b=z_b\end{cases}$

$\vec{u}\in Z:\ x_1+y_1=0\ \wedge\ x_1-y_1=z_1$
$\vec{v}\in Z:\ x_2+y_2=0\ \wedge\ x_2-y_2=z_2$

$(x_1+x_2)(y_1+y_2)=0\quad (x_1+x_2)-(y_1+y_2)=z_1+z_2$

Azpiespazio bektoriala da. $\quad x_b+y_b=0\quad x_b-y_b=z_b$

$\lambda(x,y,z)\in Z?$
$(\lambda x,\lambda y,\lambda z)\in Z$

$\lambda x+\lambda y=0\quad \lambda(x+y)=0\quad x+y=0$
$\lambda x-\lambda y=\lambda z\quad \lambda(x-y)=\lambda z\quad x-y=0$

FML-5-008

Bektore-sistema baten itxadura lineala

$F = \{(1,0,0),(0,1,0)\}$

$h(F) = h\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix} = 2 \rightarrow$ Sistema askea

$\{(1,1,0),(3,2,0),(2,3,0)\ldots\}$

Bektoreekin, konbinazio linealak bitartez, itxadura linealak sortu ditzakegu. Itxadura lineala multzo bat da.

S-ren oinarria

Azpiespazio bektoriala ← $S = \mathcal{L}(F) = \mathcal{L}(B_S)$

- S askea → S-ren oinarria
- S, lotua

S-ren sortzaileak
Sistema-sortzaileak

Bektore kopuruari kardinala

$h(B_S) = h = \dim S$

Hemen: $h(F)$ $\begin{cases} h(F)=m & \text{askea} \\ h(F)<m & \text{lotua}\end{cases}$

Bektore linealki independenteen kopurua — Bektore kopurua

$\dim S < \dim V$
↳ V-ren azpiespazio propioa da S

$\dim S = \dim V$
↳ $S = V$

$\dim S \geq 1$
↳ Dimentsioa beti izango da handiagoa edo berdina bat

$S = \{(x_1,x_2,x_3) / x_1 = 2x_2+x_3\}$ $\qquad h(F) = h\begin{pmatrix}2&1&0\\1&0&1\end{pmatrix} = 2 =$ bekt. kop

$v = (2x_2+x_3,\ x_2,\ x_3)$ $\quad u = (2,1,0) \in S$, $v = (1,0,1) \in S$ $\quad u+v \in S$? $(3,1,1)$ Bai ✓ $\quad \lambda u \in S$? $(4,2,0)$ Bai ✓

$(2x_2,x_2,0)+(x_3,0,x_3) \Rightarrow x_2(2,1,0)+x_3(1,0,1) \Rightarrow F = \{(2,1,0),(1,0,1)\}$ → Sortzailea → $S = \mathcal{L}(F)$

$T = \{(x_1,x_2,x_3) / 2x_1+x_2 = 3x_1+x_3 = 0\}$ $\quad x_1 = \frac{-x_2}{2}$ $\quad x_1 = \frac{-x_3}{3}$ $\quad x_2 = -2x_1$ $\quad x_2 = \frac{2x_3}{3}$ $\qquad \mathcal{L}(F) = T$ $\quad \dim T =$

$v = x_3(-1/3,\ 2/3,\ 1)$ $\quad$ n = 2 ekuazio → 2 ezezagun nagusi; 3 ezezagun → 1 ezezagun askea $\qquad F = \{(-1/3,\ 2/3,\ 1)\}$

$W = \{(1,m,n)\}$ ✗ $m,n \in \mathbb{R}\}$ $\qquad (1,0,0)+m(0,1,0)+n(0,0,1)$ $\qquad F = \{(1,0,0),(0,1,0),(0,0,1)\}$

$S = \{(x_1,x_2,x_3,x_4) / x_3 = 0 \wedge x_1+x_2+x_4 = 0\}$ $\qquad v = (-x_2-x_4,\ x_2,\ 0,\ x_4)$

$x_2(-1,1,0,0)+x_4(-1,0,0,1)$

$h(F) = h\begin{pmatrix}-1&1&0&0\\-1&0&0&1\end{pmatrix} = 2 =$ bekt. kop $\qquad F = \{(-1,1,0,0),(-1,0,0,1)\}$

$S = \mathcal{L}(F)$

$\dim S = 2$

M2-3-09

$$F = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$ Sistema aske edo lotua da?

$S = \underbrace{\mathcal{L}}_{\text{sistemak}}(F)$-ren oinarri bat eta dimentsioa

$\{(1,1),(0,0)\}$

$\{(0,1),(1,0)\}$

$\{(3,0),(1,0)\}$

$$h(F) = h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 1 & 0 \end{pmatrix} = 3 = \text{bektore kopurua}$$

Sistema askea

sistema eta aske = oinarria dim = 3

$$G = \left\{ \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 5 \\ -1 & 0 \end{pmatrix} \right\}$$

$$h(G) = h\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 5 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{pmatrix} = 2 \neq \text{bektore kopurua}$$

Sistema lotua

det = 0

$T = \mathcal{L}(G)$

$\dim T = 2$ $\quad B_T = \left\{ \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\} =$

V-ren oinarri bat
↳ Sistema aske

$S \subset \mathbb{R}^2$ $\quad B_S \to B_{\mathbb{R}^3}$?

$Z \in \mathbb{R}^3$ $\quad B_Z \to B_{\mathbb{R}^3}$?

B_S oinarri baten hedapena → B_V

↳ S azpiespazio baten oinarria

$S \subset V$ → Bektore-espazioa

$B_S = \{(0,1,0),(0,0,1)\} \to B'_{\mathbb{R}^3} = \{(0,1,0)\ (0,0,1)\ (1,0,0)\}$

$$h(B^1_{\mathbb{R}^3}) = h\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 3$$

$B_Z = \{(1,-1,2)\} \to B^2_{\mathbb{R}^3} = \{(1,-1,2),(0,1,0),(0,0,1)\}$

$$h(B^2_{\mathbb{R}^3}) = h\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 3$$

Hedapena → bektoreak gehitu matrize karratuak lortzeko (nahi dugun heinarekin)

FM2-J-010

1.3 Ankete

$S = \{(x_1, x_2, x_3, x_4) / x_3 = 0 \wedge x_1 + x_2 + x_3 + x_4 = 0\}$

$B_S = \{(1,-1,0,0), (0,-1,0,1)\} \Rightarrow B_{\mathbb{R}^4} = \{(1,-1,0,0), (0,-1,0,1), (0,0,1,0), (0,0,0,1)\}$

$$h(B_{\mathbb{R}^4}) = h\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 4$$

$S \subset \mathbb{R}^4 \to \dim \mathbb{R}^4 = 4$ lonenioske bekt. ke?

$B_S = \{(1,-1,2,2), (1,2,3,4)\} \to B^S_{\mathbb{R}^4} = \{(1,-1,2,2), (1,2,3,4), (0,0,1,0), (0,0,0,1)\}$

$$h(B^S_{\mathbb{R}^4}) = 4 = h\begin{pmatrix} 1 & -1 & 2 & 2 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \overset{E_2 - E_1}{=} h\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$S_T = \{(1,-1,0,2), (1,2,0,4), (3,2,0,-3)\} \to S^T_{\mathbb{R}^4} = \{(1,-1,0,2)\ (1,2,0,4)\ (3,2,0,-3), ($

$$\cancel{h(S^T_{\mathbb{R}^4}) = h\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} = h\begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 3 & 0 & 2 \\ 0 & 5 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{pmatrix}}$$

Dimenzija?

Baza, $h(S^T_{\mathbb{R}^4})$ = vekt. kprive

$$h(S^T_{\mathbb{R}^4}) = h\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \end{pmatrix} = h\begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 4 \\ 3 & 2 & -3 \end{pmatrix} = h\begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 5 & -9 \end{pmatrix} = 3$$

$B^T_{\mathbb{R}^4} = \{(1,-1,0,2), (1,2,0,4), (3,2,0,-3), (0,0,1,0)\}$

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 1 & 2 & 0 & 4 \\ 3 & 2 & 0 & -3 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

M2-5-011

1.1 Ardışık

- $B=\{x^2+x+1,\ x^2+x,\ x^2\}$ $\quad h(B_{P_2})=h\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}=3$

$\quad\quad \downarrow \quad \downarrow \quad \downarrow$

$\quad\quad \bar{e}_1 \quad \bar{e}_2 \quad \bar{e}_3$

$B=B_{P_2}$?

- $B=\{(x,1,0),(0,0,x+2),(1,1,1)\}$ $\quad x\in\mathbb{R}? \iff B'=B_{\mathbb{R}^3}$

$h(B')=h\begin{pmatrix}x&1&0\\0&0&x+2\\1&1&1\end{pmatrix}^{A_{B'}}$ $\quad \det(A_{B'})=\begin{vmatrix}x&1&0\\0&0&x+2\\1&1&1\end{vmatrix}=-(x+2)\begin{vmatrix}x&1\\1&1\end{vmatrix}=(x+2)(x-1)=0$

$x+2=0 \quad x=2$

$x-1=0 \quad x=1$

1. Kasu $\quad x\in\{-2,1\} \to h(A_{B'})=2 \quad E_f$

2. Kasu $\quad x\in\mathbb{R}-\{-2,1\} \to h(A_{B'})=3 \quad$ Bai. oinarria da

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- $S=\{(x_1,x_2,x_3,x_4)\ /\ x_1=x_2\}$, $\quad T=\mathcal{L}(\{(1,1,2,1),(2,0,-1,1)\})=\mathcal{L}(S_T)$

$\mathbb{R}^3 \longrightarrow \mathcal{L}(B_c)=\mathbb{R}^3$

$\mathcal{L}(B_c)=\mathcal{L}(\{(1,0,0),(0,1,0),(0,0,1)\})$

$\bar{v}=(x_1,x_2,x_3,x_4)=(x_1,x_1,x_3,x_4)=x_1(1,1,0,0)+x_3(0,0,1,0)+x_4(0,0,0,1)$

$h(S_S)=h\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=3=$ bekt. kop $\to$ Soistearik oinarria

$h(S_T)=h\begin{pmatrix}1&1&2&1\\2&0&1&1\end{pmatrix}=2=$ bekt. kop $\to$ soistearik oinarria

$B_S=\{(1,1,0,0),(0,0,1,0),(0,0,0,1)\}$

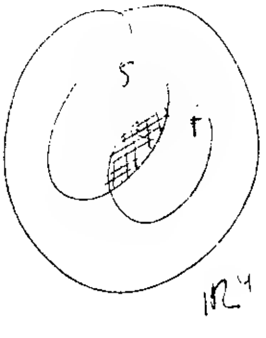
$B_T=\{(1,1,2,1),(2,0,-1,1)\}$

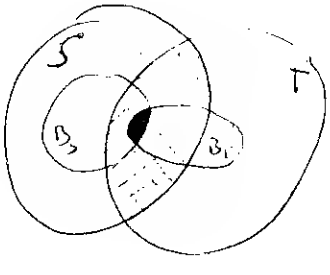
$\quad\quad \bar{v}_1{}^T \quad \bar{v}_2{}^T$

$h\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\\1&1&2&1\end{pmatrix}=h\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\\1&1&2&0\end{pmatrix}=3$

$\downarrow$

Konbinazio linealean dago, laugarren zutabea zero ezk 0 izateko baditugu.





$B_{S\cap T}=\{\bar{v}_T\}$

$h\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\\2&0&1&1\end{pmatrix}=h\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\\2&0&1&0\end{pmatrix}=4$

FM2 - S - 012

$S \subset P_3 \quad S = \{p(x) \in P_3 \;/\; p(1) = p(-1) \wedge p'(1) = 0\}$

$p(x) = ax^3 + bx^2 + cx + d$

$$\left.\begin{array}{l} p(1) = a + b + c + d \\ p(-1) = -a + b - c + d \end{array}\right\} a + \cancel{b} + c + \cancel{d} = -a + \cancel{b} - c + \cancel{d} \Rightarrow a + c = -(a+c) \Rightarrow 2(a+c) = 0$$

$p'(x) = 3ax^2 + 2bx + c$

$p'(1) = 3a + 2b + c = 0 \longrightarrow 3a + 2b + c = 2a + 2c$

$a + 2b - c = 0$

$2a = -2c$

$a = -c \quad c = -a$

$3a + 2b - a = 0$

$2a = -2b$

$a = -b$

$-a = b$

$-3c + 2b - c = 0$

$-4c = 2b$

$-2c = b$

$p(x) = -cx^3 - 2cx^2 + cx + d$

$p(x) = ax^3 - ax^2 - ax + d$

$a(x^3 - x^2 - x) + d$

$\Updownarrow$

$(d, -a, -a, a)$

$\Updownarrow$

$d(1,0,0,0) + a(0,-1,-1,1) \Leftrightarrow (d,0,0,0) + (a,-a,-a,a)$

$h(B_S) = h\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix} = 2 \qquad B_S = \{1, -x - x^2 + x^3\}$

$(V, <,>) \longrightarrow \mathbb{R}^n, P_n, M_{m \times n}(\mathbb{R}), C, [a,b)$

V: bektorea

$<,>$: biderketa eskalarra $\longrightarrow$
- norma euklidearra (bektore batena) $*_1$
- distantzia (bi bektoreen artekoa) $*_2$
- proiekzioa ($\bar{x}, \bar{y}$ gainean) $*_3$ $\longrightarrow$ ortogonalketa (orainarte hutsean) $\downarrow$ biderketa lineal hobea

$<x, x^2> = \int_{-1}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4}\left[1^4 - (-1)^4\right] = 0$

$*_1 \quad \|\bar{x}\| = +\sqrt{<\bar{x}, \bar{x}>}$

$*_2 \quad \|\bar{x} - \bar{y}\| = d(\bar{x}, \bar{y}) = +\sqrt{<(\bar{x}-\bar{y}), (\bar{x}-\bar{y})>}$

$*_3 \quad P_{\bar{y}}\bar{X} = \frac{<\bar{x}, \bar{y}>}{\|\bar{y}\|^2} \cdot \bar{y}$

$\cos\theta = \frac{<\bar{x}, \bar{y}>}{\|\bar{x}\| \cdot \|\bar{y}\|} \qquad \cos\theta = \frac{16}{\sqrt{14} \cdot \sqrt{21}} = \frac{16}{7\sqrt{6}}$

Adib

$\bar{x} = (1, 2, 3)$

$\bar{y} = (2, 1, 4)$

$\|\bar{x}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$ $\qquad \bar{x} - \bar{y} = (-1, 1, -1)$

$\|\bar{y}\| = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21}$

$d(\bar{x}, \bar{y}) \Rightarrow \sqrt{(-1)^2 + 1^2 + (-1)^2} = \sqrt{3}$

$P_{\bar{y}}\bar{X} \Rightarrow \frac{16}{21} \cdot (2,1,4) \Rightarrow \left(\frac{32}{21}, \frac{16}{21}, \frac{64}{21}\right)$

$P_{\bar{x}}\bar{y} \Rightarrow \frac{16}{14} \cdot (1,2,3) \Rightarrow \frac{8}{7}(1,2,3) \Rightarrow \left(\frac{8}{7}, \frac{16}{7}, \frac{24}{7}\right)$

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FM2-I-013

OSNARRI baten ORTOGONALKETA

$B_S (\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_r) \;/\; \mathcal{L}(B_S) = S$ → azpiespazio bektoriala

$B_{og} (\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_r)$ → $\mathcal{L}(B^S_{og}) = S$

$B_{og} = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_r\}$

Bektoreak binaka hartuz, elkarren arteko biderketa eskalarrak zero izan behar du.

B_C: Oinarri ortogonalak

$\mathbb{R}^3$: $B_C = \{(1,0,0), (0,1,0), (0,0,1)\}$

$$\begin{array}{ccc} (1,0,0) & (1,0,0) & (0,1,0) \\ (0,1,0) & (0,0,1) & (0,0,1) \\ \hline 0 & 0 & 0 \end{array}$$

GRAM-SCHMIDT prozedura

$\bar{u}_1 = \bar{v}_1$

$\bar{v}_2 = \bar{u}_2 - \dfrac{\langle \bar{u}_2, \bar{v}_1 \rangle}{\|\bar{v}_1\|^2} \cdot \bar{v}_1 = \bar{u}_2 - P_{v_1} \bar{u}_2$

$\bar{v}_3 = \bar{u}_3 - \dfrac{\langle \bar{u}_3, \bar{v}_1 \rangle}{\|\bar{v}_1\|^2} \bar{v}_1 - \dfrac{\langle \bar{u}_3, \bar{v}_2 \rangle}{\|\bar{v}_2\|^2} \cdot \bar{v}_2$

$= \bar{u}_3 - P_{v_1} \bar{u}_3 - P_{v_2} \bar{u}_3$

$\bar{v}_4 = \bar{u}_4 - P_{v_1} \bar{u}_4 - P_{v_2} \bar{u}_4 - P_{v_3} \bar{u}_4$

$B_S = \{(1,2,0), (3,0,4) \;/\; S = \mathcal{L}(B_S)\}$

Ortogonala da? $1 \cdot 3 + 2 \cdot 0 + 0 \cdot 4 = 3$ Ez, ez da ortogonala
duelako ortogonala

$B_{og} = (\bar{v}_1, \bar{v}_2) = (\bar{u}_1, \bar{v}_2)$ $\quad \bar{u}_1 = \bar{v}_1$ $\quad B_{og} = \{(1,2,0), (\frac{12}{5}, -\frac{6}{5}, 4)\}$

$\bar{v}_2 = (3,0,4) - \dfrac{3}{5} \cdot (1,2,0) = (3,0,4) - (\frac{3}{5}, \frac{6}{5}, 0) = (\frac{12}{5}, -\frac{6}{5}, 4)$

$S \cap T$
$S \cup T$

$S \Rightarrow B_S = \{(1,1,0,0), (0,0,1,0), (0,0,0,1)\}$ (bektoreak: $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$) $\qquad S \cap T = \bar{0}$ edo $S \cap T \neq \bar{0}$

$T \Rightarrow B_T = \{(1,1,2,1), (2,0,1,-1)\}$ (bektoreak: $\bar{v}_1$, $\bar{v}_2$)

$B_{S \cap T} = \{w_1, w_2, \ldots\} = \{\bar{v}_1\} = \{(1,1,2,1)\}$

$\bar{u}_1 \in T$? $\quad h(\bar{v}_1, \bar{v}_2, \bar{u}_1) = h\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \end{pmatrix} = 3 \Rightarrow$ ez dago konbinazio linealean

$\bar{u}_2 \in T$? $\quad h(\bar{v}_1, \bar{v}_2, \bar{u}_2) = h\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = 3$

$\bar{v}_1 \in S$? $\quad h(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{v}_1) = h\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 \end{pmatrix} = 3 \Rightarrow$ Konbinazio linealean dago

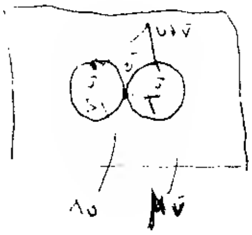
FH2 · 5 · 014

Dim orokorren bektore kopurua

S / $B_S = \{(\overset{\bar{v}}{1,2,0})\}$ dim S = 1

T / $B_T = \{(\overset{\bar{v}}{1,0,1})\}$ dim T = 1

$S \cap T = \bar{0} = (0,0,0)$



$\bar{u}+\bar{v} \in S \cup T$? $\bar{u}+\bar{v} = (2,2,1)$ Ez, konpontzeko, erdian $\lambda\bar{u}$ eta $\mu\bar{v}$ notatzen.

Nola kalkulatu zenbait bateko bektore baten koordenatuak

↓

ELS → Forma matriziala

ELS Forma bektoriala

$$\begin{cases} x+2y+3z=4 \\ x-y+z=2 \\ x \quad -z=1 \end{cases} \quad A^* = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

$$x(1,1,1)+y(2,-1,0)+z(3,1,-1)=(4,2,1)$$

forma bektoriala

$\bar{u}_1=(1,1,1)$, $\bar{u}_2=(0,-1,2)$

$\bar{K}=(-2,5,-4) \in \mathbb{R}^3$, $\bar{u}_1$ eta $\bar{u}_2$ konb. lineal. bezala

$$\lambda_1\bar{u}_1+\lambda_2\bar{u}_2=\bar{K}$$

$$\lambda_1(1,1,1)+\lambda_2(0,-1,2)=(-2,5,-4)$$

$$A^*=\begin{pmatrix} 1 & 0 & -2 \\ 1 & -1 & 5 \\ 1 & 2 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$h(A^*)=2$

$h(A)=2$

$$\sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{pmatrix}$$

$\lambda_1 = 2$

$\lambda_2 = -3$

↓

$2(-1,1,1)+-3(0,-1,2)$

$(-2,2,2)+(0,3,-6)$

$(-2,5,-4)$

M2-3 c15

$x\bar{a}_1 + y\bar{a}_2 + z\bar{a}_3 = \bar{b}$

$A_{+} = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 0 & 3 & 4 \\ 3 & 1 & 4 & 5 \end{pmatrix}$ $\quad A\bar{x} = \bar{b} \quad \begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

$x(1,2,3) + y(1,0,-1) + z(2,3,4) = (3,4,5)$

$B_S = \{\bar{a}_1, \bar{a}_2, \bar{a}_3, \ldots\}$

$\downarrow$

$h(B_S) = $ | zuk. sistema. .. | bektore. h(B) = m

$\downarrow$

Din. bekt. kop.

ELS BATERAEZINA SOLUZIO HURBILDUA

$x_1 + x_3 = 1$
$x_2 + 2x_3 = 1$
$x_1 + 2x_2 + 2x_3 = 1$
$x_1 + x_2 + 3x_3 = 1$

$A_{+} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{pmatrix}$

$h(A) \neq h(A_+)$

$\{(1,0,1,1), (0,1,2,1), (1,2,2,3)\}$

$B_S^3 = x_1(1,0,1,1) + x_2(0,1,2,1) + x_3(1,2,2,3)$

$x_1\bar{a}_1 + x_2\bar{a}_2 + x_3\bar{a}_3 = \bar{b}$

1) $B_S \rightarrow$ oinarria
2) $B_S \rightarrow B_{oz}$
3) $\bar{b}$-ren $H L(S) = \bar{b}'$
4) $A\bar{x} = \bar{b}' \longrightarrow$ S. bateragarria
5) Sistema ebatzi $\bar{x}$: sol. hurbildua

$h(B_S^3) = 3$ oinarria da

GRAM-SCHMIDT $\Rightarrow B_{oz} = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$

$\bar{v}_1 = \bar{u}_1 = (1,0,1,1)$

$\bar{v}_2 = \bar{u}_2 - P_{v_1}\bar{u}_2 = \bar{u}_2 - \frac{\langle \bar{u}_2, \bar{v}_1 \rangle}{\|v_1\|^2}\bar{v}_1 = \bar{u}_2 - \frac{3}{3}v_1 = \bar{u}_2 - \bar{v}_1$

$\bar{v}_3 = \bar{u}_3 - P_{v_1}\bar{u}_3 - P_{v_2}\bar{u}_3 = \bar{u}_3 - \frac{6}{3}\bar{v}_1 - \frac{3}{3}\bar{v}_2 = \bar{u}_3 - 2\bar{v}_1 - \bar{v}_2$

$\langle \bar{u}_2, \bar{v}_1 \rangle = 3$
$\langle \bar{u}_3, \bar{v}_1 \rangle = 6$
$\langle \bar{u}_3, \bar{v}_2 \rangle = 3$
$\|\bar{v}_1\|^2 = 3$
$\|\bar{v}_2\| = 3$

$\bar{v}_2 = (0,1,2,1) - (1,0,1,1) \quad (-1,1,1,0)$

$\bar{v}_3 = (1,2,2,3) - (2,0,2,2) - (-1,1,1,0) = (0,1,-1,1)$

$B_{oz} = \{(1,0,1,1), (-1,1,1,0), (0,1,-1,1)\}$

FM2-F-016

$A\bar{x}=\bar{b}$ $\bar{b}$ cerca del sistema $\overset{=}{\bar{b}'}$

$\bar{b}'=\sum_{i=1} P_{\bar{v}_i}\bar{b} = P_{\bar{v}_1}\bar{b}+P_{\bar{v}_2}\bar{b}+P_{\bar{v}_3}\bar{b}$

$\bar{b}=(1,1,1,1)$ $\qquad \frac{\langle\bar{b},\bar{v}_1\rangle}{\|\bar{v}_1\|^2}\bar{v}_1+\frac{\langle\bar{b},\bar{v}_2\rangle}{\|\bar{v}_2\|^2}\bar{v}_2+\frac{\langle\bar{b},\bar{v}_3\rangle}{\|\bar{v}_3\|^2}\bar{v}_3$

$\langle\bar{b},\bar{v}_1\rangle=3$

$\langle\bar{b},\bar{v}_2\rangle=1$

$\langle\bar{b},\bar{v}_3\rangle=1$

$\|v_1\|^2=3$

$\|v_2\|^2=3$

$\|v_3\|^2=3$

$\frac{3}{3}\bar{v}_1+\frac{1}{3}\bar{v}_2+\frac{1}{3}\bar{v}_2$

$\bar{v}_1+\frac{1}{3}(\bar{v}_2+\bar{v}_3)$

$(1,0,1,1)+\frac{1}{3}(-1,2,0,1)$

$(1,0,1,1)+(-1/3,2/3,0,1/3)$

$(2/3,2/3,1,4/3)$

$A\cdot\bar{x}=\bar{b} \Rightarrow A\bar{x}=\bar{b}'$

$$A_Z^{\star}=\begin{pmatrix}1&0&1&\vdots&2/3\\0&1&2&\vdots&4/3\\1&2&2&\vdots&1\\1&1&3&\vdots&4/3\end{pmatrix}\qquad h(A)=h(A_Z)=3\qquad \begin{pmatrix}1&0&0&\vdots&1/3\\0&1&0&\vdots&0\\0&0&1&\vdots&1/3\end{pmatrix}\quad \begin{matrix}x_1=1/3\\x_2=0\\x_3=1/3\end{matrix}$$

$h(A)\le 3$

$h(A_A)\le 4$ } SBs → Det $h=3$; → S.Indet [redacted] $h=2$

↓ $h(A)=h(A_A)$

$e=d(\bar{b},\bar{b}')=\|\bar{b}-\bar{b}'\|$

$=\|(1/3,1/3,0,-1/3)\|$

$=+\sqrt{(1/3)^2+(1/3)^2+(-1/3)^2}$

$=+\sqrt{\frac{1+1+1}{9}}=+\sqrt{\frac{3}{9}}=+\sqrt{1/3}$

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det A → A i es polinomio característico

$|A|$

$|A-\lambda I|=0$ $\quad / A\in M_{n\times n}(\mathbb{R})$; $\sigma(A)=\{\lambda_1(k_1),\lambda_2(k_2),\dots\}$

$k_1+k_2+\dots=n$

$(-1)^n\cdot P_n(\lambda)=(-1)^n\cdot(\lambda-\lambda_1)^{k_1}\cdot(\lambda-\lambda_2)^{k_2}\dots$

$|A-\lambda I|_{\lambda=0}\Rightarrow|A|=[(-1)^n\cdot P_n(\lambda)]_{\lambda=0}$

A:3

$\sigma(A)=\{1(k=1),-2(k=1),3(k=1)\}$ $\quad / A\in M_{3\times3}(\mathbb{R})$

$P(\lambda)=[(\lambda-1)(\lambda+2)(\lambda-3)]_{\lambda=0}$ $\quad |A|=(-1)(-1)(2)(-3)=-6$

[redacted]

$\sigma(B)=\{2(k=2),-3(k=1)\}$ → $\{2,2,-3\}$

$(-1)^3P(\lambda)=(-1)^3[(\lambda-2)^2(\lambda+3)]_{\lambda=0}$

$|A|=12\quad(-1)(-2)^2\,3$

2-1-017

Eigenvalues [ ]
Eigenvectors [ ]
Eigensystem [ ]

$D_{4\times4}$ / $\sigma(A) = \{1, -3, 2, 0\}$

$(-1)^4 P(\lambda) = [1\,(\lambda-1)(\lambda+3)(\lambda-2)(\lambda-0)]_{\lambda=0}$

$|A| = 1\cdot(-1)\cdot 3\cdot(-2)\cdot 0 = 0$

$$S = \left\{ \begin{pmatrix} a+b+3c & 2a-b \\ -a-c & a+2b+5c \end{pmatrix} / a,b,c \in \mathbb{R} \right\}$$

$a(1,2,-1,1) + b(1,-1,0,2) + c(3,0,-1,5)$

$$h(G_S) = h\begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & -1 & 0 & 2 \\ 3 & 0 & -1 & 5 \end{pmatrix} = h\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -3 & 1 & 1 \\ 0 & -6 & 2 & 2 \end{pmatrix} = h\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -3 & 1 & 1 \\ \cancel{0} & \cancel{-3} & \cancel{1} & \cancel{1} \end{pmatrix} = h$$

Bases but eten dimentsion kalkulate

$$G_S = \left\{ \underset{\bar{e_1}}{\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}}, \underset{\bar{e_2}}{\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}}, \underset{\bar{e_3}}{\begin{pmatrix} 3 & 0 \\ -1 & 5 \end{pmatrix}} \right\} \qquad B_S = \{\bar{e_1}, \bar{e_2}\}$$

$a\bar{e_1} + b\bar{e_2} + c\bar{e_3}$

$\bar{u} = (1, 2, -1)$
$\bar{v} = (0, 1, 3)$

$<\bar{u},\bar{v}> = 0+2-3 = -1$

$\|\bar{u}\| = +\sqrt{<\bar{u},\bar{u}>} = \sqrt{1^2+2^2+(-1)^2} = \sqrt{6}$

$\|\bar{v}\| = \sqrt{0+1+9} = \sqrt{10}$

$d(\bar{u},\bar{v}) = \|\bar{u}-\bar{v}\| = \|(1,1,-4)\|$

$P_{\bar{v}}\,\bar{u} = \frac{<\bar{u},\bar{v}>}{\|v\|^2}\cdot \bar{v}$

$P_{\bar{u}}\,\bar{v} =$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -1-\lambda & 0 & -1 \\ 0 & 0 & -1-\lambda & -1 \\ 0 & -1 & -1 & -2-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -1-\lambda & 0 & -1 \\ 0 & -1-\lambda & -1 \\ -1 & -1 & -2-\lambda \end{vmatrix} = (-1)^3(1-\lambda)\begin{vmatrix} 1+\lambda & 0 & 1 \\ 0 & 1+\lambda & 1 \\ 1 & 1 & 2+\lambda \end{vmatrix}$$

<<(never) forget orthogonalization!
per Characteristic Polynomial

$[(-1-\lambda)(-1-\lambda)(-2-\lambda)] - [(-1-\lambda)+(-1-\lambda)]$

$(1+\underbrace{\lambda+\lambda}_{2\lambda}+\lambda^2)(-2-\lambda) - (-2-2\lambda)$

$-2-4\lambda-2\lambda^2-\lambda-2\lambda^2-\lambda^3+2+2\lambda$

$-\lambda^3-4\lambda^2-3\lambda$

$\Downarrow$

$(1-\lambda)(-\lambda^3-4\lambda^2-3\lambda)$

$(1+\lambda)(1+\lambda)(2+\lambda) - (1+\lambda) - (1+\lambda)$

$(1+\lambda+\lambda+\lambda^2)(2+\lambda) - 2(1+\lambda)$

$\cancel{2}+4\lambda+2\lambda^2+\lambda+2\lambda^2+\lambda^3\cancel{-2}-2\lambda$

$\lambda^3+4\lambda^2+3\lambda$

$\Downarrow$

$(\lambda-1)(\lambda^3+4\lambda^2+3\lambda)$

$(\lambda-1)\left[(1+\lambda)^2(2+\lambda) - (1+\lambda) - (1+\lambda)\right] \Rightarrow (\lambda-1)(1+\lambda)\left[(1+\lambda)(2+\lambda) - 2\right]$

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$$(\lambda-1)(1+\lambda)\left[\lambda^2+3\lambda+2-2\right] \Rightarrow (\lambda-1)(\lambda+1)\,\lambda\,(\lambda+3)=0$$

$$\boxed{\begin{array}{l}\lambda=1\\ \lambda=-1\\ \lambda=0\\ \lambda=-3\end{array}}$$

ELS - HOMOGJENE

↓

Forma matriciale

$$(A-\lambda I)\,X_{n\times1}=(0)_{n\times1} \qquad X=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$$

1) GAUSS

2) ROUCHE-FROB $\longrightarrow \left\{\begin{array}{l}\text{Gr. neg}\\ \text{Gr. esk} \longrightarrow \dim S(A)\end{array}\right.$

3) OSNARRI BAT KALKULATU

$$(A_+)_{\lambda=0}=\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&1&0\\0&0&1&1&0\\0&1&1&2&0\end{array}\right)$$

$$\sim\left(\begin{array}{cccc|c}1&0&0&0&0\\0&1&0&1&0\\0&0&1&1&0\end{array}\right)$$

$$h(A)=h(A_+)=3<4\left\{\begin{array}{l}\text{Gr. neg.}\to 3\\ \text{Gr. eske}\to 1\end{array}\right.$$

$x_1=0$

$x_2+x_4=0$

$x_3+x_4=0$

$x_1=0$

$x_2=x_3=-x_4$

dimentsioa $\rightarrow 1$